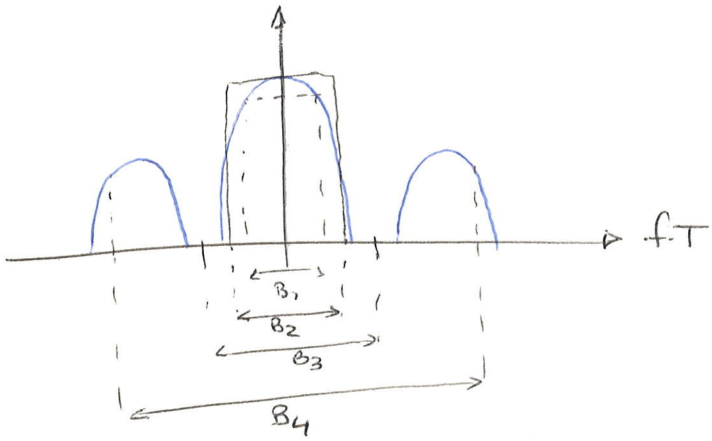
TEMA 4: SISTEMAS DE TRANSMISIÓN DIGITAL PASO BANDA

4.1 - INTRODUCCIÓN

Vamos a ver modulaciones ⇒ antenas más eficientes y multiplexación de información

Anchura de banda: distintas formas de medirlo

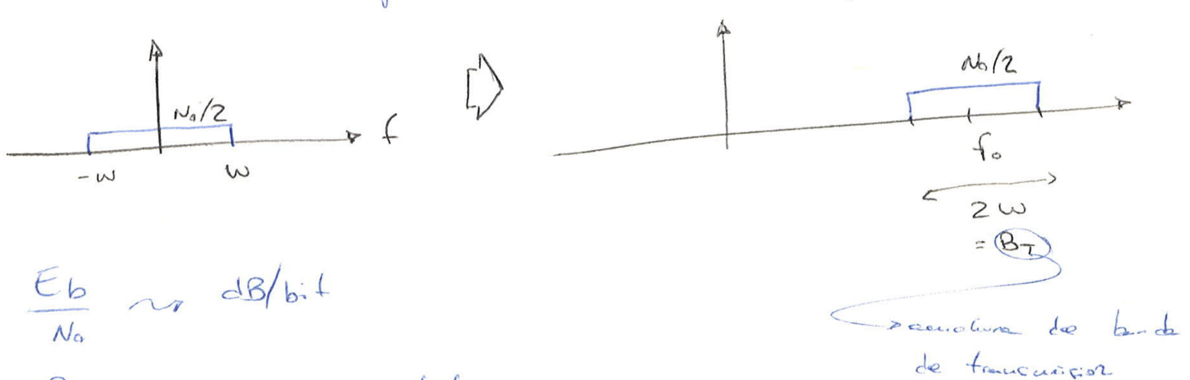
- A 3dB (B_1)
- Equivalente de ruido (B_2)
- De primeros nulos (B_3)
- Contenido de potencia fraccional (B_4)



Probabilidad de error: en función de la SNR

$$\boxed{SNR = \frac{S}{N} = \frac{E_b \cdot R}{\frac{N_0}{2} \cdot 2B_T} = \frac{E_b}{N_0} \cdot \frac{R}{B_T}}$$

E_b = energía por bit (J/bit)



$\frac{E_b}{N_0}$ ~ dB/bit

$\frac{R}{B_T}$ = eficiencia espectral

$$\boxed{\eta = \frac{R\ (bps)}{B_T\ (Hz)}}$$

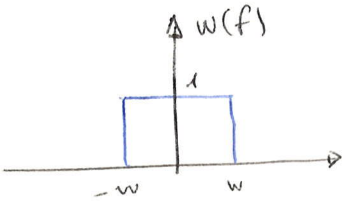
Objetivos:

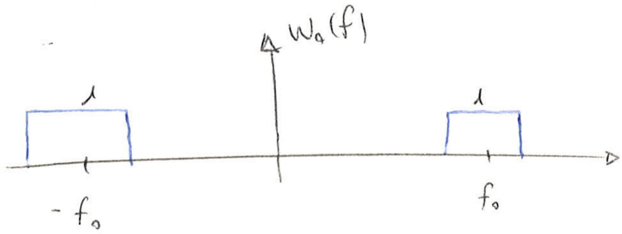
1.- Minimizar la probabilidad de error por bit (BER)
2.- Minimizar la potencia (P) requerida
3.- Maximizar la eficiencia espectral (η)
4.- Minimizar la complejidad

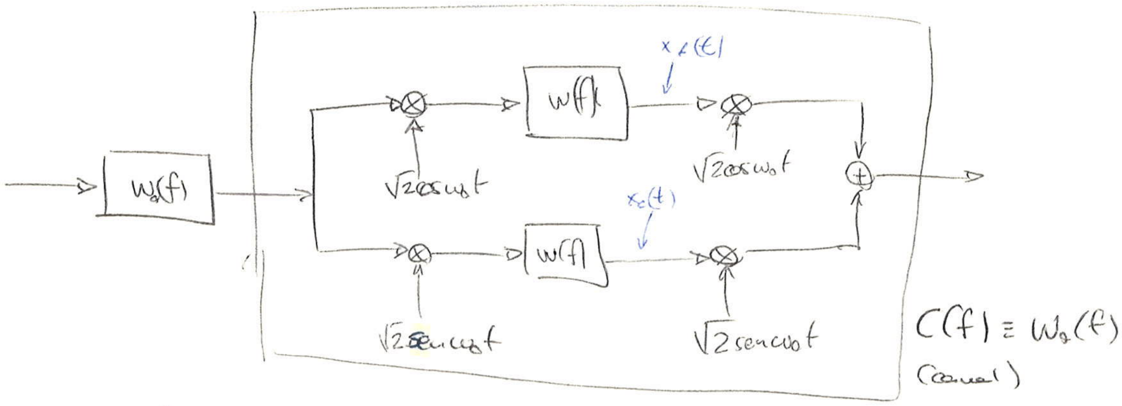
Objetivos incompatibles entre sí.

4.2- PLANTEAMIENTO GENERAL

Blau blau blau... Señal analítica y equivalente paso bajo... zzzzzz

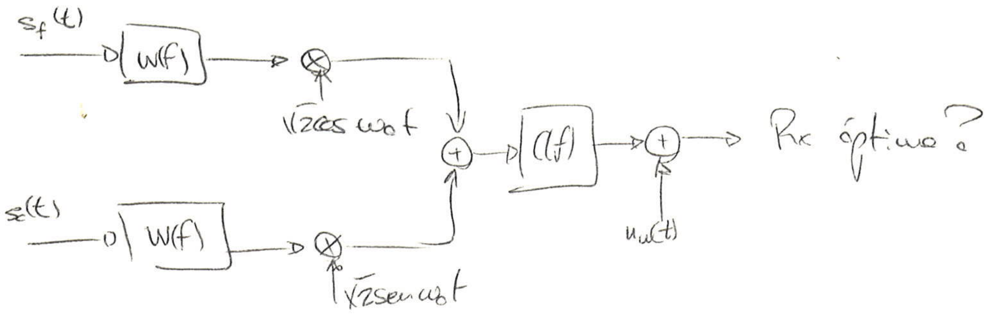






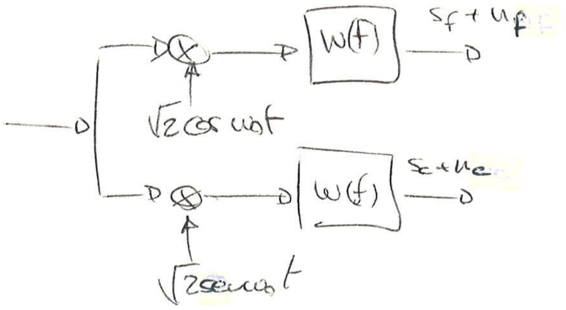
$$x(t) = \left(x_f(t)\cos\omega_0 t + x_c(t)\operatorname{sen}\omega_0 t\right)\sqrt{2}$$

Es posible recuperar la información sólo si consigo sincronizar.



$n_w(t) = (n_f(t)\cos\omega_0 t + n_c(t)\,\text{sen}\,\omega_0 t)\sqrt{2} + n_{fb}(t)$

fuera de banda



Conclusiones:

1.- Tenemos un canal doble (fase/cuadratura)

2.- No hay que cambiar la teoría para el Rx óptimo

3.- El ruido fuera de banda es irrelevante (lo filtramos)

4.- Hay que garantizar el sincronismo

$b(t)$ equivalente paso bajo

$x(t) = \text{Re}\left(b(t)\cdot e^{j\omega_0 t}\right)$

Tema 1 $\rightarrow$ $y(t) = \sum_n A(nT_s)\cdot b(t - nT_s)$

$y(t) = \text{Re}\left(\sum_n A_k(nT_s)\, b(t - nT_s)\cdot e^{j\omega_0 t}\right)$

Sólo vale para PAM $\rightarrow$ $B_k(nT_s) + j\,C_n(nT_s)$

4.3- Técnicas de Modulación Digital

4.3.1 - PAM

(Pulse Amplitude Modulation)

4.3.1.1 - ASK-DSB

$$b(t) = b^R(t) + j\, b^I(t)$$

En ASK-DSB, $b^I(t) = 0$

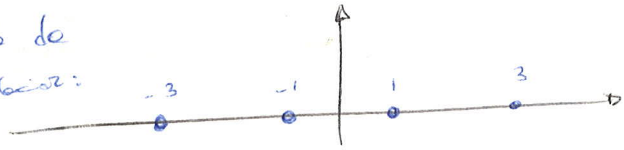
$$A_{km}(mT_s) \in \mathbb{R}$$

$$A_{km} = B_k = \frac{2k-1}{2} \cdot d \qquad k \in \left\{-\frac{M}{2}+1, \dots, \frac{M}{2}\right\}$$

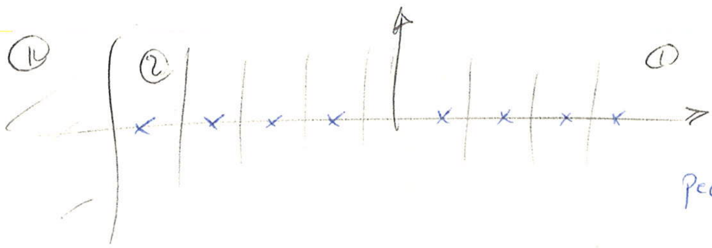
$$y(t) = \mathrm{Re}\left(\sum_m B_k(mT_s) \cdot b^R(t-mT_s) \cdot e^{j\omega_0 t}\right) =$$

$$= \left(\sum_m B_k(mT_s) \cdot b^R(t-mT_s)\right) \cos \omega_0 t$$

Ejemplo de constelación:



$M = 4$



$$P_{e①} = Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$P_{e②} = 2Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$\mathcal{E} = \frac{d^2}{12}(M^2 - 1)$$

$\mathcal{E} = \mathcal{E}(d)$

$d = d(\mathcal{E})$

$$p_e = 2\left(1-\frac{1}{M}\right) Q\left(\sqrt{\frac{6}{M^2-1}\,\frac{E}{N_0}}\right)$$

E = energía media por símbolo

$$E = E_b \cdot \log_2 M$$

$$p_e = 2\left(1-\frac{1}{M}\right) Q\left(\sqrt{\frac{6}{M^2-1}\,\frac{E_b\cdot\log_2 M}{N_0}}\right)$$

$$\boxed{\eta = \frac{R}{B_T} = \frac{2W\cdot\log_2 M}{2W} = \log_2 M \ \text{(bps/Hz)}}$$

$r_{max} = 2\cdot W$ (caso ideal)

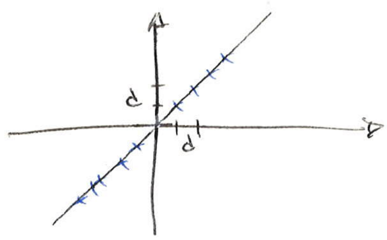
$R = r\cdot M$

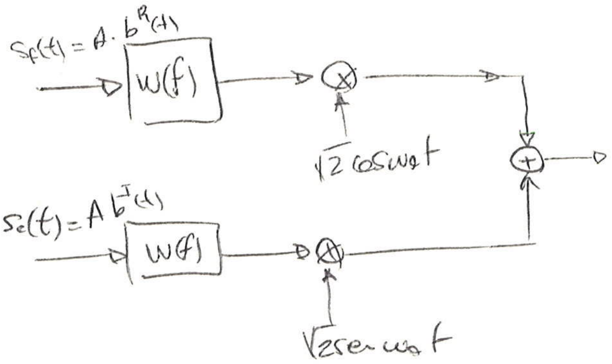
$M\uparrow \Rightarrow \left\{\begin{array}{l}\eta\uparrow\\ p_e\uparrow\end{array}\right. \rightarrow$ compromiso

4.3.1.2 - ASK-SSB

$$b^I(t) = TH\{b^R(t)\}$$

Usa menos banda pero el sistema es más complejo.





Ahora la distancia entre señales es $d\sqrt{2}$

$$p_e = 2\left(1-\frac{1}{M}\right) Q\left(\frac{d/\sqrt{2}}{\sqrt{N_0/2}}\right)$$

$$\boxed{p_e = 2\left(1-\frac{1}{M}\right) Q\left(\sqrt{\frac{3}{M^2-1}\cdot\frac{E_b}{N_0}\cdot 2\log_2 M}\right)}$$

$$\boxed{\eta = \frac{2W\log_2 M}{W} = 2\log_2 M \ \text{(bps/Hz)}}$$

4.3.1.3 - QASK

Q = Quadrature

$$y(t) = \mathrm{Re}\left(\sum_m A_{kn}(mT_s) \cdot b(t-mT_s) \cdot e^{j\omega_0 t} \right)$$

Suponemos $b(t)$ real y $A_{kn}(mT_s)$ complejo:

$$A_{kn}(mT_s) = B_k(mT_s) + j\,C_n(m \cdot T_s)$$

Hay M símbolos equiprobables ($\sqrt{M}$ niveles)

$$\begin{cases} B_k = \dfrac{2k-1}{2} d \\ C_n = \dfrac{2n-1}{2} d \end{cases} \qquad k, n \in \left\{ -\frac{\sqrt{M}}{2}+1, \dots, \frac{\sqrt{M}}{2} \right\}$$

Constelación cuadrada:

$$P_e = p(e/m_{①}) \cdot p_{①} + p(e/m_{②}) p_{②} + p(e/m_{③}) p_{③}$$

$$p(e/m_{①}) = 2Q\left(\frac{d/2}{\sqrt{N_0/2}}\right) - Q^2(\dots) \qquad [\text{esquinas}]$$

$$p(e/m_{②}) = 3Q(\dots) - 2Q^2(\dots) \qquad [\ \square\ ]$$

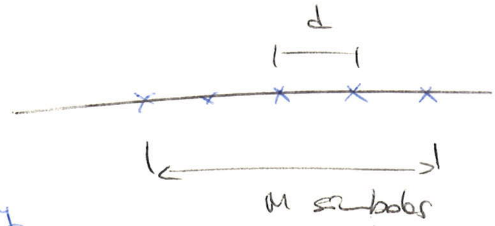
$$p(e/m_{③}) = 4Q(\dots) - 4Q^2(\dots) \qquad [\ \square\ ]$$

$$p_{①} = \frac{4}{M} \qquad p_{②} = \frac{4(\sqrt{M}-2)}{M} \qquad p_{③} = \frac{(\sqrt{M}-2)^2}{M}$$

$$P_e = \frac{4}{M}(2Q - Q^2) + \frac{4(\sqrt{M}-2)}{M}(3Q - 2Q^2) + \frac{(\sqrt{M}-2)^2}{M}(4Q - 4Q^2)$$

Ahora ponemos la distancia en función de la energía. De ASK_DSB:

$$E = \frac{d^2}{12}(M^2-1)$$



Ahora en horizontal hay $\sqrt{M}$ símbolos

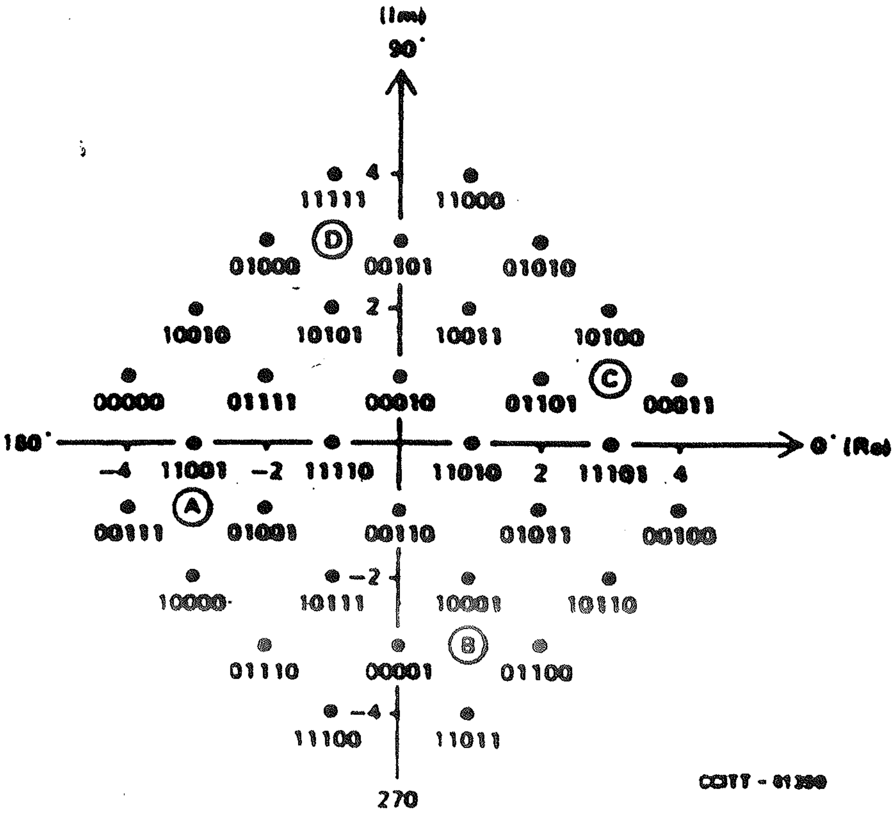
$$E = \frac{d^2}{6}(M-1) \to \frac{d/2}{\sqrt{N_0/2}} = \sqrt{\frac{3}{M-1}\frac{E}{N_0}}$$

$$p_e = 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)\left(1 - \underbrace{\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)}_{\text{Si SNR} \uparrow \text{, esto es despreciable}}\right)$$

$$\to \boxed{p_e \simeq 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E}{N_0}}\right)}$$

$E = E_b \log_2 M$

La pe de símbolo: $p_e \simeq 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_b}{N_0}\log_2 M}\right)$



Los números binarios representan $Y0_n$ $Y1_n$ $Y2_n$ $Q3_n$ $Q4_n$

FIGURA 3/V.32

Constelación de señal de 32 puntos con codificación en rejilla para 9600 bit/s y subconjunto de estados A, B, C y D utilizados a 4800 bit/s y para el acondicionamiento

Combinaciones de modulaciones de amplitud y de fase

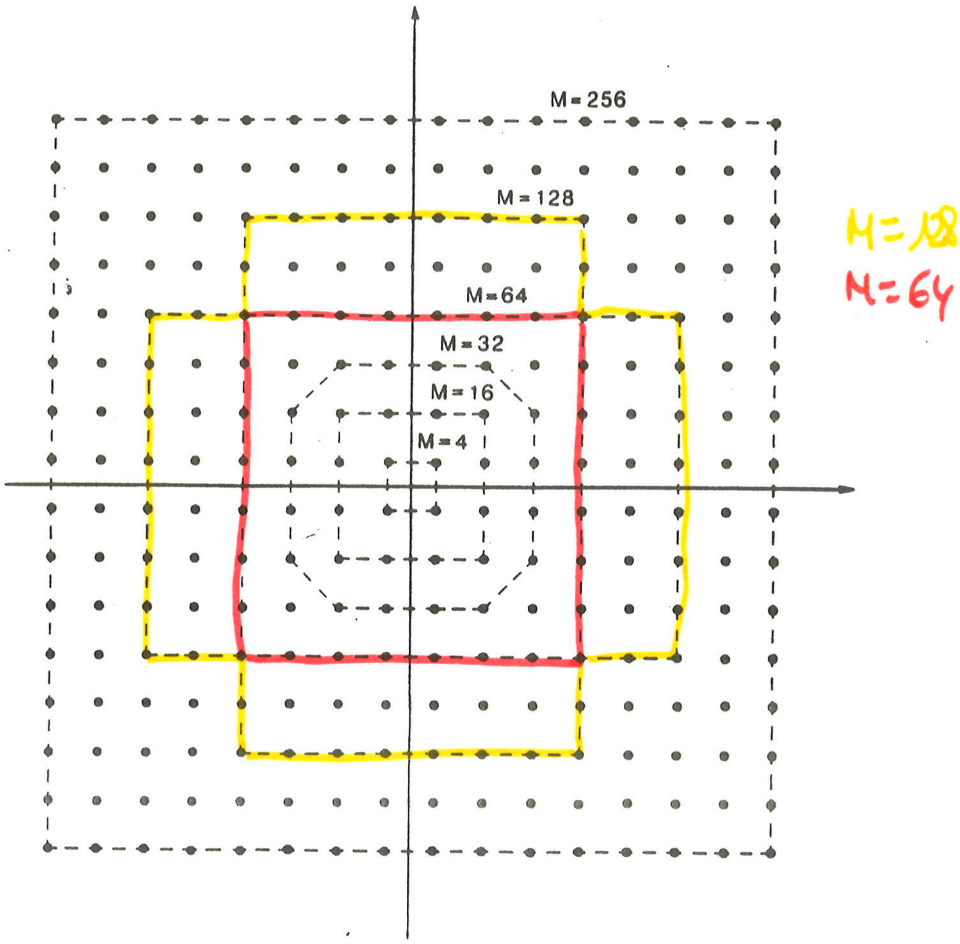


Figure 5.22 Rectangular AM–PM signal constellations. The dotted lines delimit the region of the signal points pertaining to that particular M. Notice the cross constellations for $M = 32 = 2^5$ and $M = 128 = 2^7$.

$$\boxed{\eta = \frac{R(bps)}{B_T(Hz)} = \frac{r \cdot \log_2 M}{B_T} = \log_2 M}$$

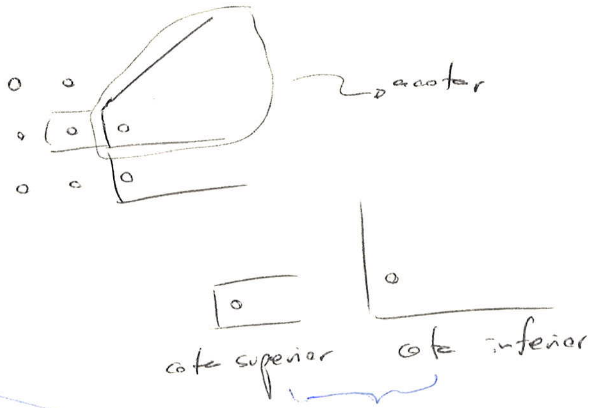
¿por qué?

→ en banda base, $r/B_T = 2$, al modular B_T se duplica ⇒ $r/B_T = 1$ (idealmente)

4.3.1.5 - ASK+PSK (AM-PM)

(transparencia rejilla)

Seguimos dando vueltas:



→ funciona mejor que la de Arthurs...

4.3.1.4 - PSK (ooooolé)

$$\boxed{A_{kn} = \sqrt{E} \cdot e^{j\frac{2\pi k}{M}}}$$

$$Q\left(\frac{\sqrt{E}\,\operatorname{sen}\pi/M}{\sqrt{N_0/2}}\right) < p_e < 2Q\left(\frac{\sqrt{E}\,\operatorname{sen}\pi/M}{\sqrt{N_0/2}}\right)$$

$$Q\left(\operatorname{sen}\pi/M\sqrt{\frac{2E_b}{N_0}\log_2 M}\right) < p_e < 2Q\left(\operatorname{sen}\frac{\pi}{M}\sqrt{\frac{2E_b}{N_0}\log_2 M}\right)$$

pe de símbolo

$$\boxed{\frac{p_e}{\log_2 M} \leq BER \leq p_e} \qquad \boxed{\eta = \log_2 M \ (\text{bps/Hz})}$$

4.3.2- FSK

$$s_k(t) = \sqrt{\frac{2E}{T}}\, b(t)\cdot\cos(\omega_0 + \omega_k)t \qquad 0 \leq t \leq T$$

$$\langle b^2(t)\rangle = 1$$

$$\omega_k = 2\pi f_k \quad , \quad f_k = \frac{2k-1}{2} f_d$$

$$k \in \left\{-\frac{M}{2}+1, \dots, \frac{M}{2}\right\}$$

$$\left.\begin{array}{l}\cos\omega t\\ \cos(\omega+\Delta\omega)t\end{array}\right\} \quad 0 = \int_0^T \cos\omega t\cdot\cos(\omega+\Delta\omega)t\,dt$$

ortogonales

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$A = (2\omega + \Delta\omega)t$$
$$B = \Delta\omega t$$

$$0 = \int_0^T \cos(2\omega + \Delta\omega)t \, dt + \int_0^T \cos \Delta\omega t \, dt$$

$$\Rightarrow \cancel{\frac{\text{sen}(2\omega + \Delta\omega)T}{2\omega + \Delta\omega}} \rightarrow 0 + \frac{\text{sen}\, \Delta\omega T}{\Delta\omega} = 0$$

$2\omega + \Delta\omega$ es muy grande

$$\text{sen}\, \Delta\omega T = 0 \iff \Delta\omega T = k \cdot \pi \quad k \in \mathbb{Z}$$

$$\boxed{\Delta f \cdot T = k/2}$$

$\Rightarrow$ ortogonalidad de dos tonos, siempre que la diferencia de fases sea cero.

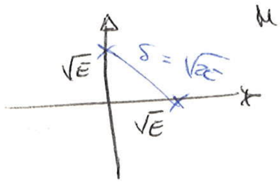
$$\boxed{f_k = \frac{2k-1}{2} f_d, \quad f_d T = \frac{k}{2}}$$

$$P_e \leq (M-1) \cdot p_e[M=2]$$

para una constelación ortogonal.

$$p_e[M=2] = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

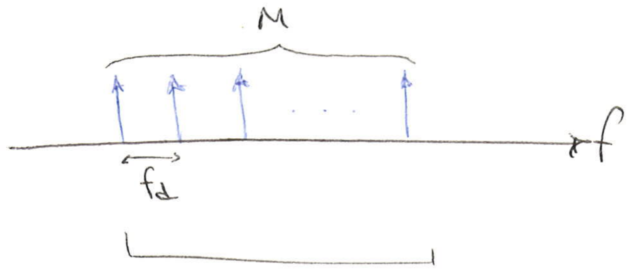
$M=2$



$$P_e \leq (M-1) \cdot Q\left(\sqrt{\frac{E}{N_0}}\right)$$
$$= (M-1) \cdot Q\left(\sqrt{\frac{E_b}{N_0} \log_2 M}\right)$$

Eficiencia espectral:

$$\eta = \frac{R(bps)}{B_T}$$



$$B_T = (M-1)\, f_d \simeq M\cdot f_d$$

$$\boxed{\eta \simeq \frac{\frac{1}{T}\cdot \log_2 M}{M\cdot f_d} = \frac{1}{f_d\cdot T}\cdot\frac{\log_2 M}{M}}$$

En el mejor caso posible, $K = 1 \rightarrow f_d\cdot T = 1/2 \rightarrow$

$$\rightarrow \boxed{\eta \simeq \frac{2\log_2 M}{M} \text{ bps/Hz}}$$

Si $M\uparrow$ $\begin{cases} \eta \downarrow \\ E\uparrow,\ pe\downarrow \end{cases}$

Para una constelación binaria, FSK ortogonal no es lo mejor posible.

<u>Ejemplo:</u>

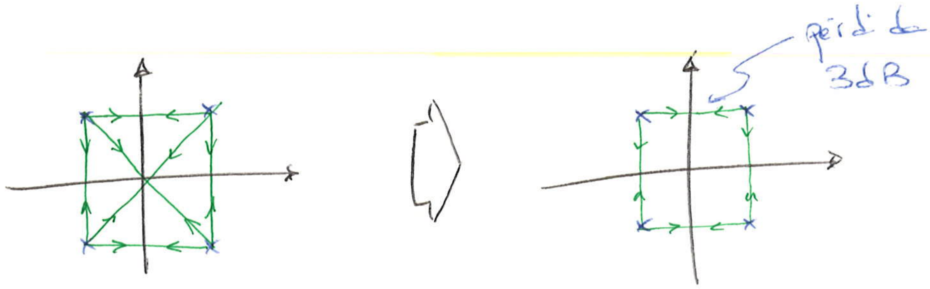
Averiguar la distancia entre 2 tonos según su diferencia de frecuencias.

4.3.3- OTRAS TÉCNICAS

4.3.3.1 - OPSK

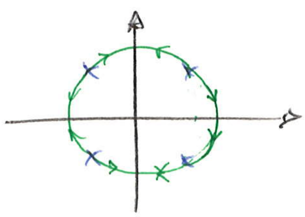
O ≡ offset

Prevenir las transiciones de 180° (que implican que en algún momento la potencia se hace cero).



4.3.3.2 - MSK (Minimum Shift Keying)

Se le da una forma a las señales para que las transiciones sean circulares



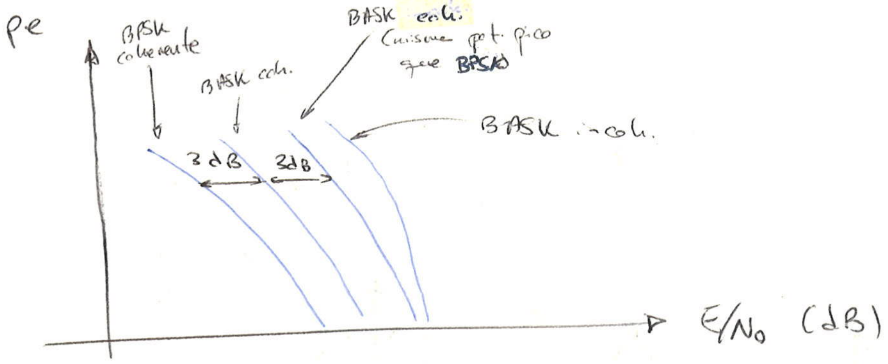
4.3.3.3 - DPSK

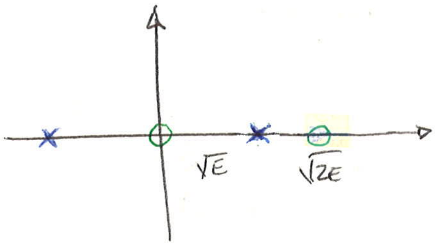
Resolver el problema de la determinación de la fase, transmitiendo la información en los cambios de fase.

Las prestaciones son algo inferiores a PSK, pero permite la transmisión en canales no coherentes. Son inferiores porque, al ser un sistema con memoria, los errores se propagan.

4.4- Prestaciones

4.4.1- Comparación de sistemas

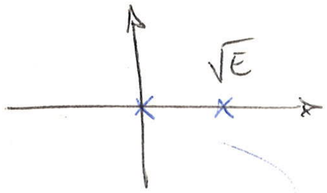




× → BPSK $\quad p_e = Q\left(\sqrt{\frac{2E}{N_0}}\right)$

o → BASK $\quad p_e = Q\left(\sqrt{\frac{E}{N_0}}\right)$

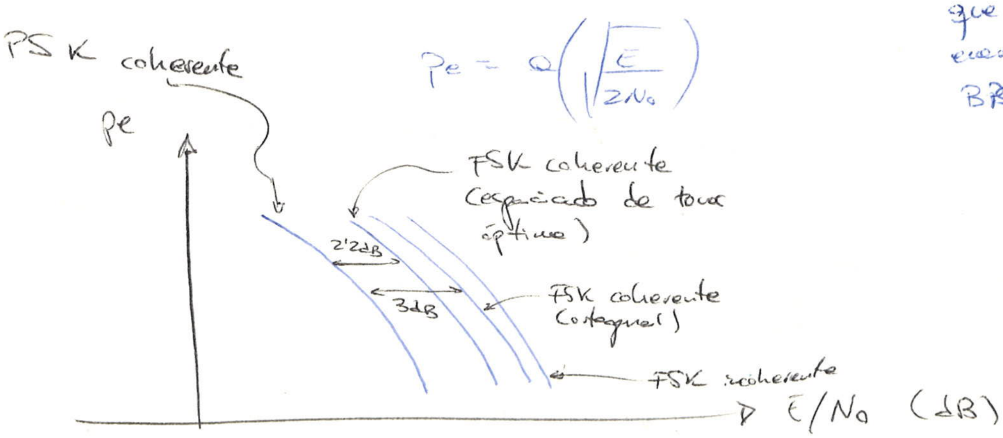
← BASK con la misma energía media es 3dB peor



← BASK con la misma potencia de pico ($\sqrt{E}$)

→ símbolos 3dB más cerca ⇒ sistema 3dB peor que BASK con misma energía media que BASK

$$P_e = Q\left(\sqrt{\frac{E}{2N_0}}\right)$$



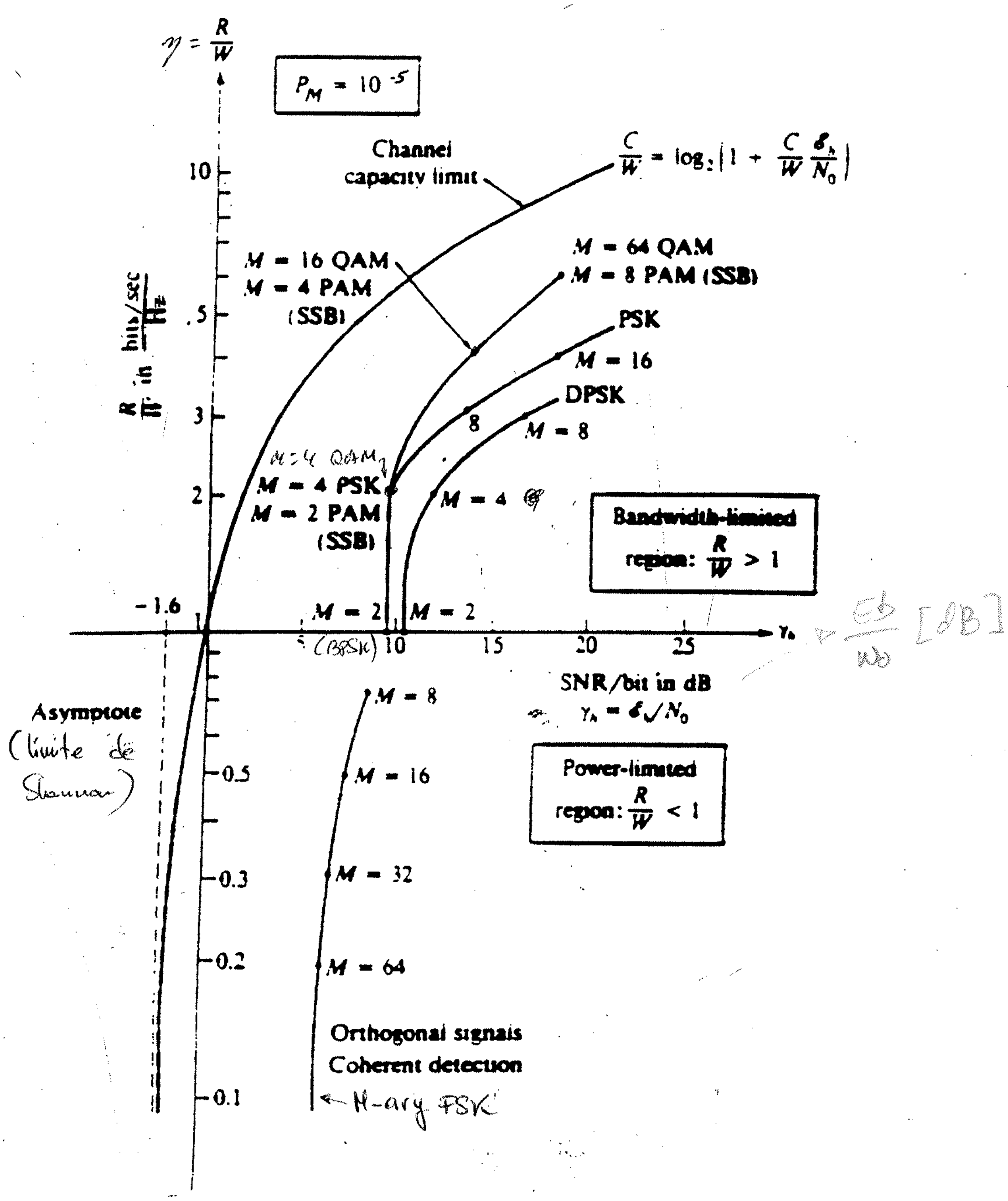


FIGURE 4.2.29
Comparison of several modulation methods at 10^{-5} symbol error probability.

LA transparencia

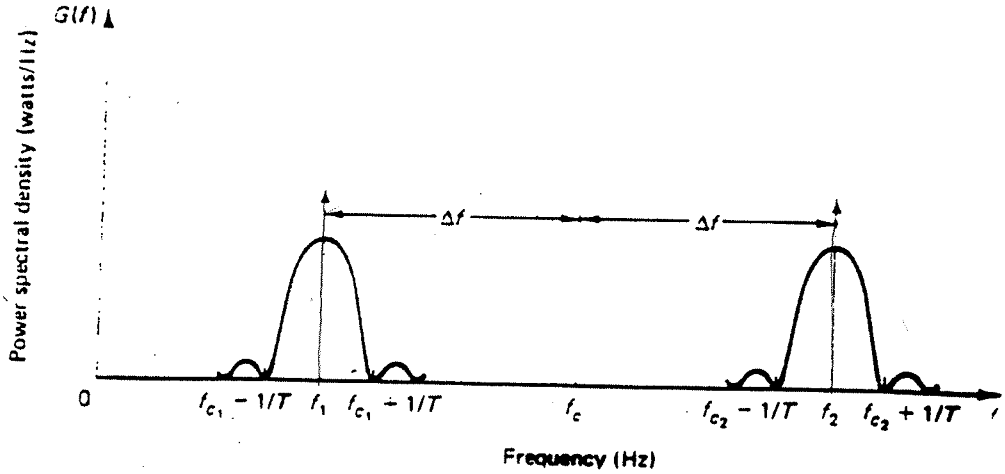


Figure 8.49 Power spectral density for BFSK.

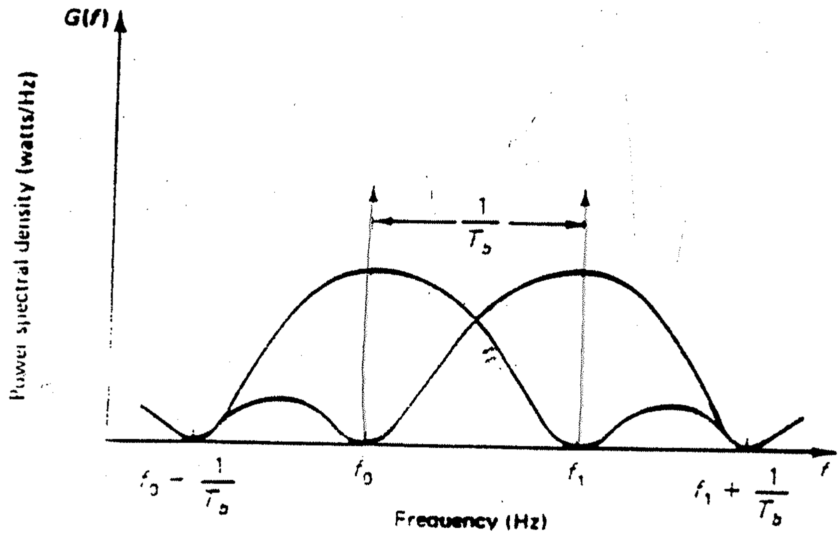


Figure 8.50 Power spectral density for BFSK, orthogonal tone spacing.

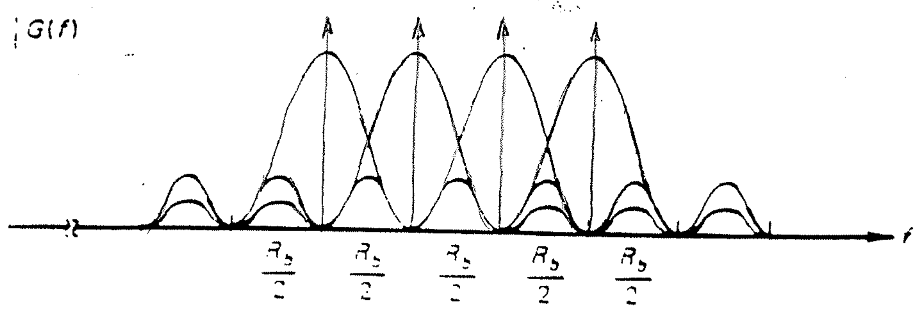


Figure 8.56 Power spectral density for 4-ary FSK.

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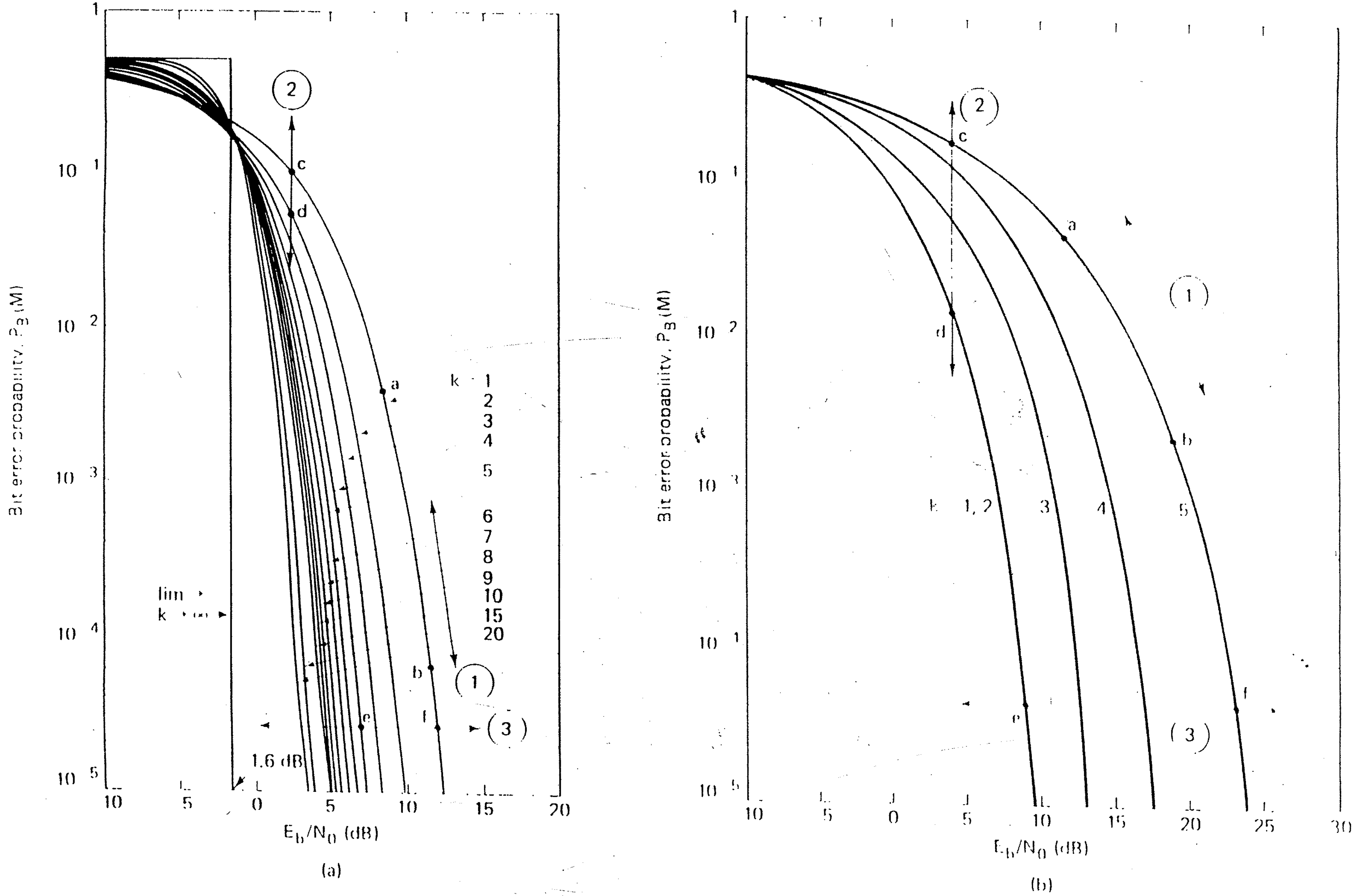
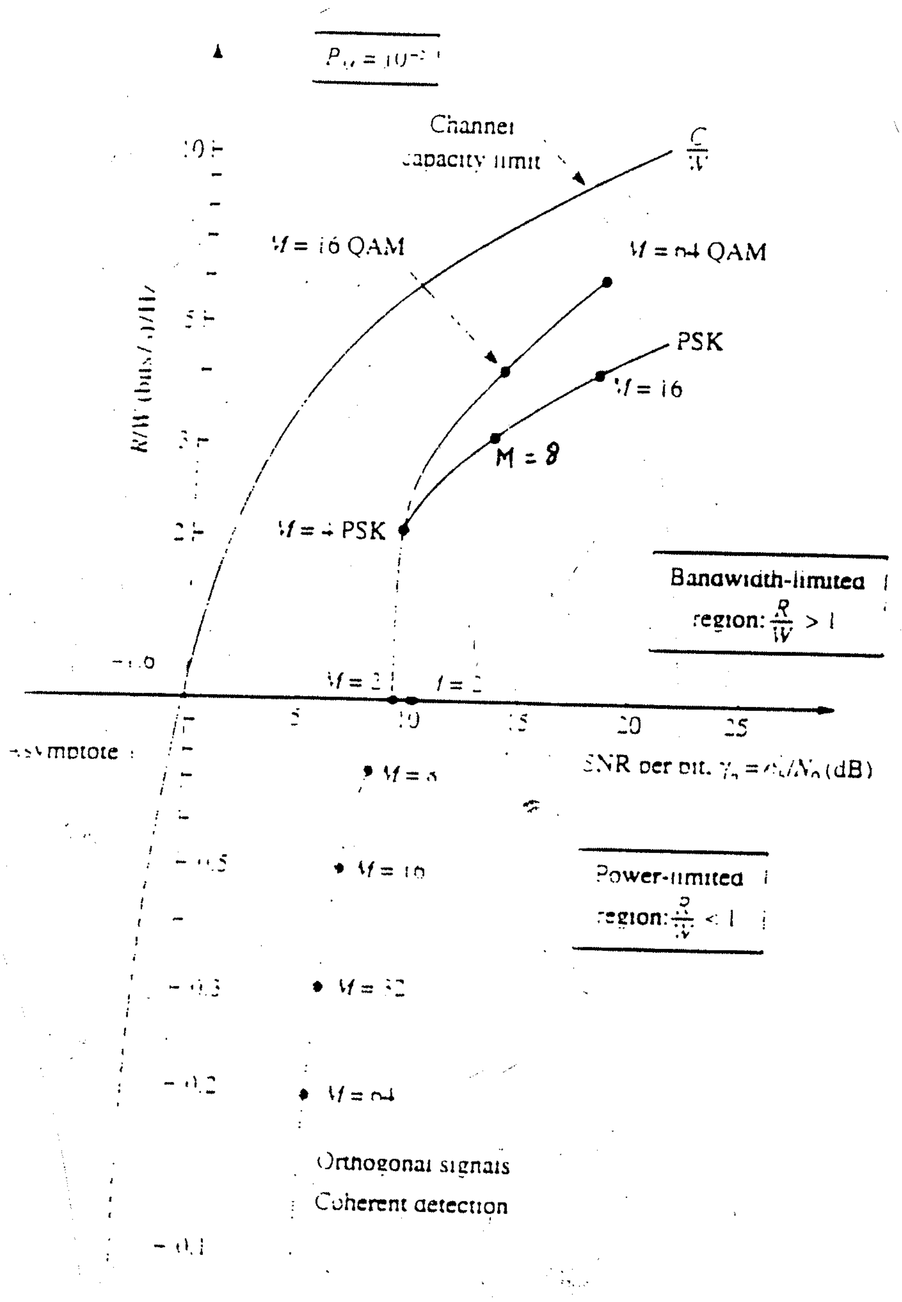


Figure 7.1 Bit error probability versus E_b/N_0 for coherently detected M ary signaling. (a) Orthogonal signaling. (b) Multiple phase signaling.



2-17 Comparison of several modulation methods at 10^{-5} symbol error probability

4.4.2- Criterios de Diseño

- Coste
- Complejidad
- R (bps)
- BER máxima
- B_T máxima
- P_T máxima
- P_{Rx}
- Tiempo de adquisición máximo del Rx

Problemas

① ¿Puede formarse una señal ASK sobre una portadora de 1MHz mediante la mezcla de la información con una onda periódica triangular de 500kHz? ¿Y de 200kHz?

Los armónicos pares no aparecen ⇒ no se puede.

Si $x(t-T/2) = -x(t)$, los armónicos pares no aparecen.

$$X(f)\cdot e^{-j2\pi f\frac{T}{2}} = -X(f)$$

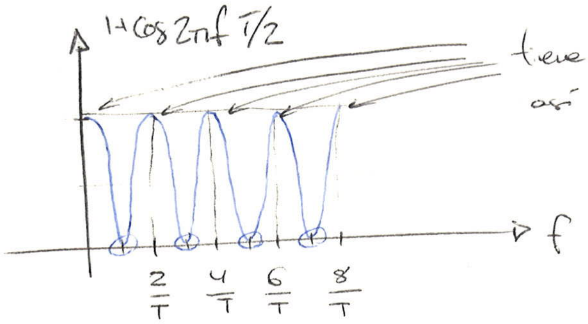
$$\Rightarrow X(f)\left(1 + e^{-j2\pi f\frac{T}{2}}\right) = 0$$

módulo

$$\Rightarrow \left|1+\cos 2\pi f T/2 - j\,\mathrm{sen}\, 2\pi f T/2\right| = \sqrt{2(1+\cos 2\pi f T/2)}$$

110

$X(f) \cdot \sqrt{2(1+\cos 2\pi f T/2)} \cdot e^{j\varphi} = 0$



tiene que ser cero el producto, así que $X(f)$ es cero

Si la portadora es de 200kHz, el 5° armónico sí lo podemos sacar.

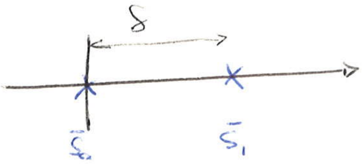
(2) a) Receptor óptimo de complejidad mínima

$s_0(t) = 0 \qquad s_1(t) = \text{sen}(1000t + \vartheta)$

Canal AWGN, $p_0 = 1/4$

b) Hallar pe si $N_0/2 = 10^{-3}$ W/Hz y $R = 100$ bps

c) ¿BW de primer nulo?



Posición de $\bar{s}_1$:

100 bps = 100 señales/s $\rightarrow$ ~~T = 0'01s~~

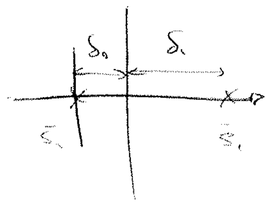
$\neq 1$, ya que $p_0 \neq p_1$

$R = r(\text{baudios}) \cdot H (\text{bits/símbolo}) \qquad H = p_0 \log_2 \frac{1}{p_0} + p_1 \log_2 \frac{1}{p_1} =$

$100 = r \cdot 0'81 \Rightarrow r = 123'26$ baudios $\qquad = 0'81$ bits/símbolo

$= 1/T$

$T = 0'0081$ $\qquad \boxed{\delta^2 = T/2}$



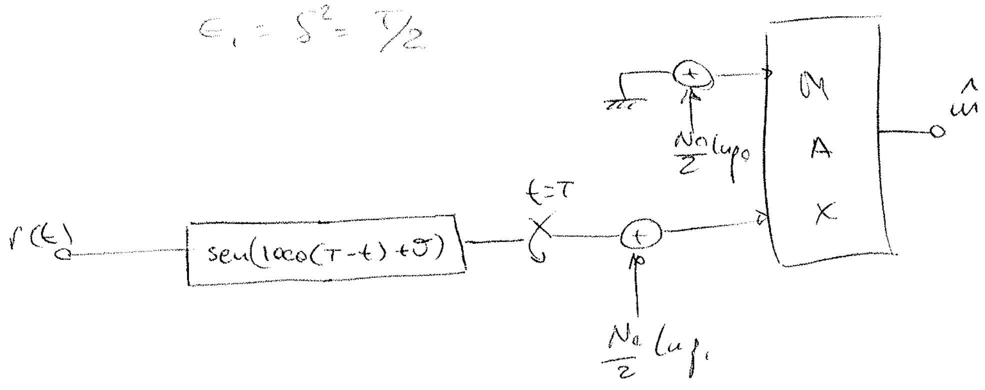
$$p_e = p_0 \cdot Q\left(\frac{\delta_0}{\sqrt{N_0/2}}\right) + p_1 \cdot Q\left(\frac{\delta_1}{\sqrt{N_0/2}}\right)$$

$$\begin{cases} \delta_0 = \dfrac{\delta}{2} + \dfrac{N_0}{2\delta} \ln \dfrac{p_0}{p_1} \\[2ex] \delta_1 = \dfrac{\delta}{2} + \dfrac{N_0}{2\delta} \ln \dfrac{p_1}{p_0} \end{cases}$$

a) $\hat{m} = \max_i \left((\vec{r}, \vec{s}_i) - \frac{1}{2} E_i + \frac{N_0}{2} \ln p_i \right)$

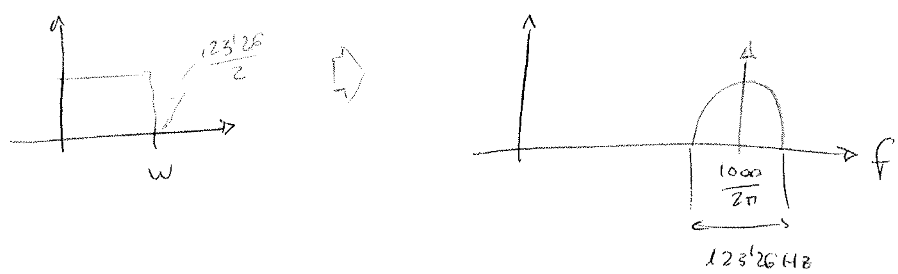
$E_0 = 0$

$E_1 = \delta^2 = T/2$



c) $r = 123'26$ baudios

$r_{max} \to 2W$ en banda base



(4) Mensajes equiprobables, canal AWGN

$s_1(t) = \cos \omega t \qquad s_2(t) = \cos(\omega + \Delta\omega)t$

$T = 1ms$

$f = 1MHz$

$\Delta f = 250 Hz$

$N_0/2$ tal que $E/N_0 = 6$

Calcular p_e. También si $\Delta f = 500 Hz$

$$\int_0^T \cos \omega t \cos(\omega + \Delta\omega) t \, dt \underset{\uparrow}{\simeq} \frac{T}{2} \operatorname{sinc} 2 \Delta f T = 0$$

despreciando el término de frecuencia doble

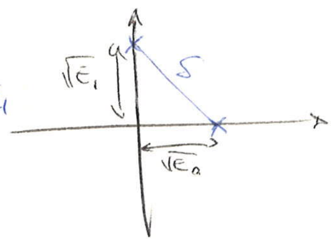
$\Rightarrow 2\Delta f T = K \qquad \Delta f \cdot T = K \qquad K \in \mathbb{Z}$

$\Delta f_1 = 250 Hz \Rightarrow \Delta f \cdot T = 1/4 \longrightarrow$ señales no ortogonales

$\Delta f_2 = 500 Hz \Rightarrow \Delta f \cdot T = 1/2 \longrightarrow$ ortogonales

Caso 2:

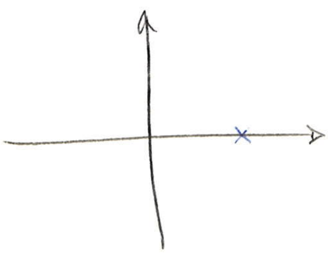
$E_0 = \frac{1}{2} \cdot 1ms = 5 \cdot 10^{-4} = E_1$
$= E$



$$p_e = Q\left(\frac{\delta/2}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

$$= Q(\sqrt{6})$$

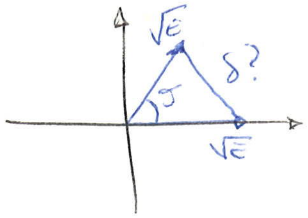
Caso 1:



Pa casa
(Gram-Schmidt)

$$(x,y) = \|x\|\cdot\|y\|\cdot\cos\vartheta$$
$$= \sqrt{\tfrac{T}{2}}\sqrt{\tfrac{T}{2}}\cdot\cos\vartheta = \tfrac{T}{2}\,\mathrm{sinc}\,2\Delta f T$$

$$\cos\vartheta = \mathrm{sinc}\,2\Delta f T$$
$$= \mathrm{sinc}\,1/2 = \frac{\mathrm{sen}\,\pi/2}{\pi/2} = 2/\pi$$



Teorema del coseno: $\delta^2 = E + E - 2E\cdot\cos\vartheta$
$$= 2E\cdot\left(1 - 2/\pi\right)$$

etc.

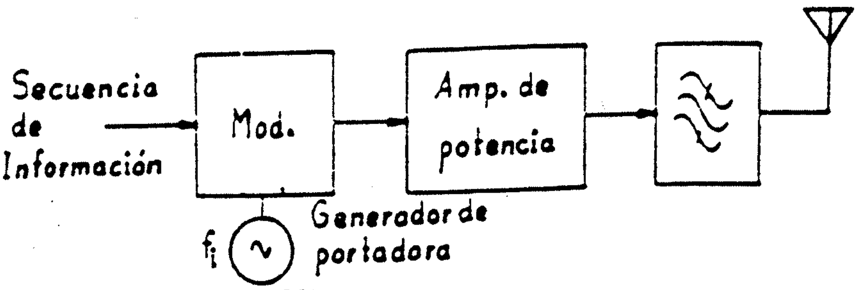


Figura 8.11. Transmisor PSK.

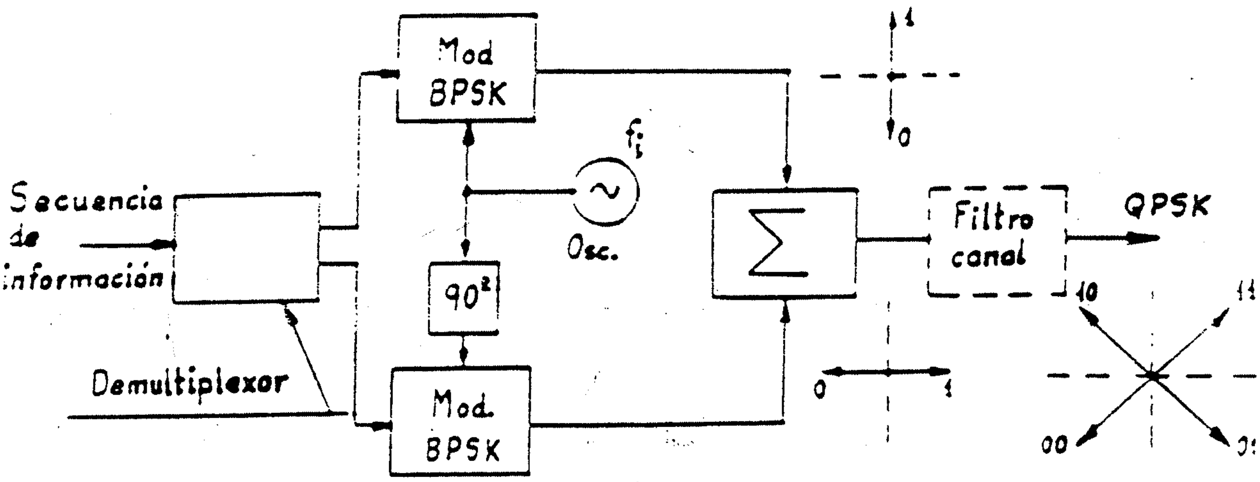


Figura 8.13. Modulador QPSK.

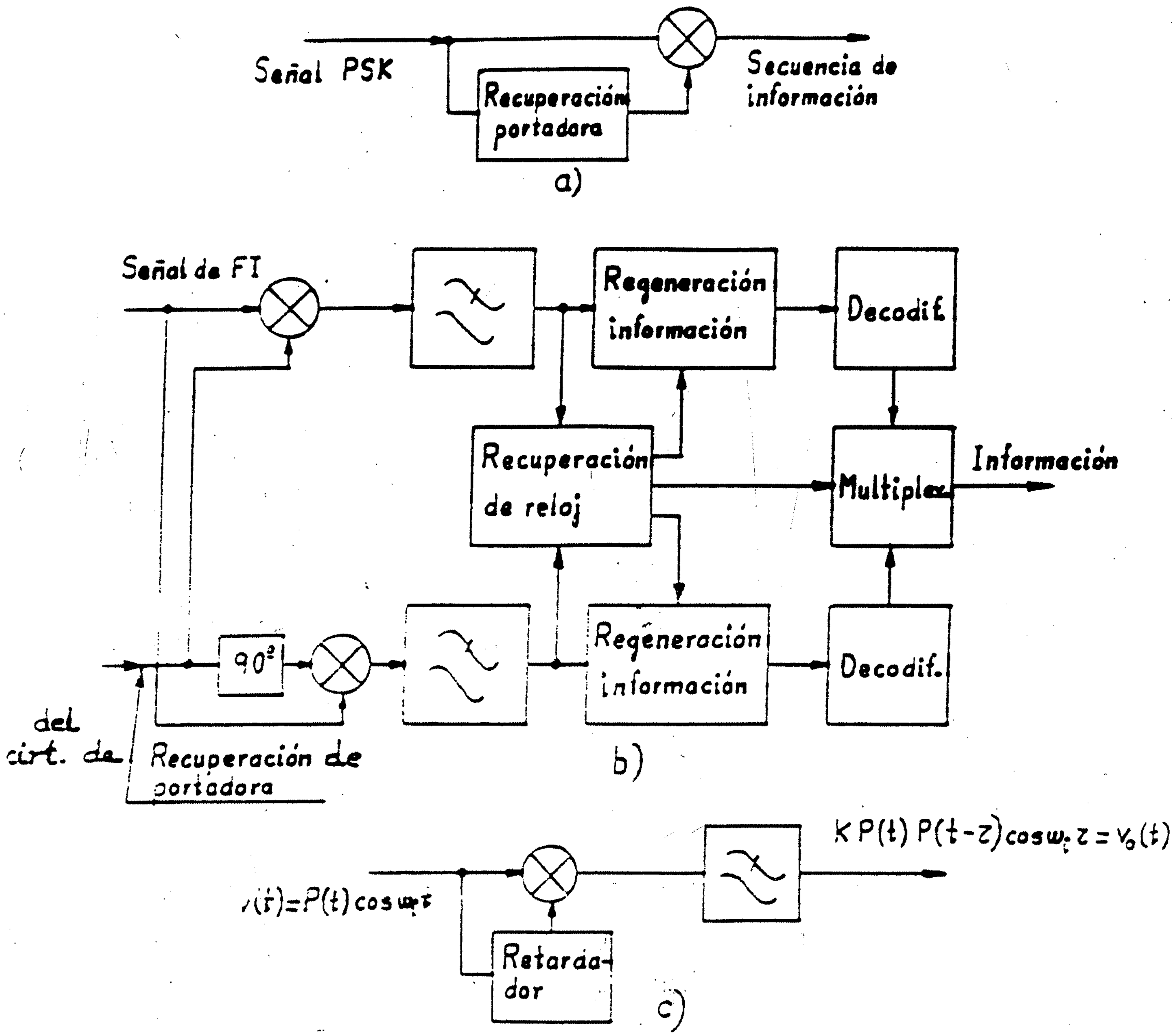


Figura 8.16. a) Detector coherente de BPSK
b) Detector coherente de QPSK
c) Detector Diferencial

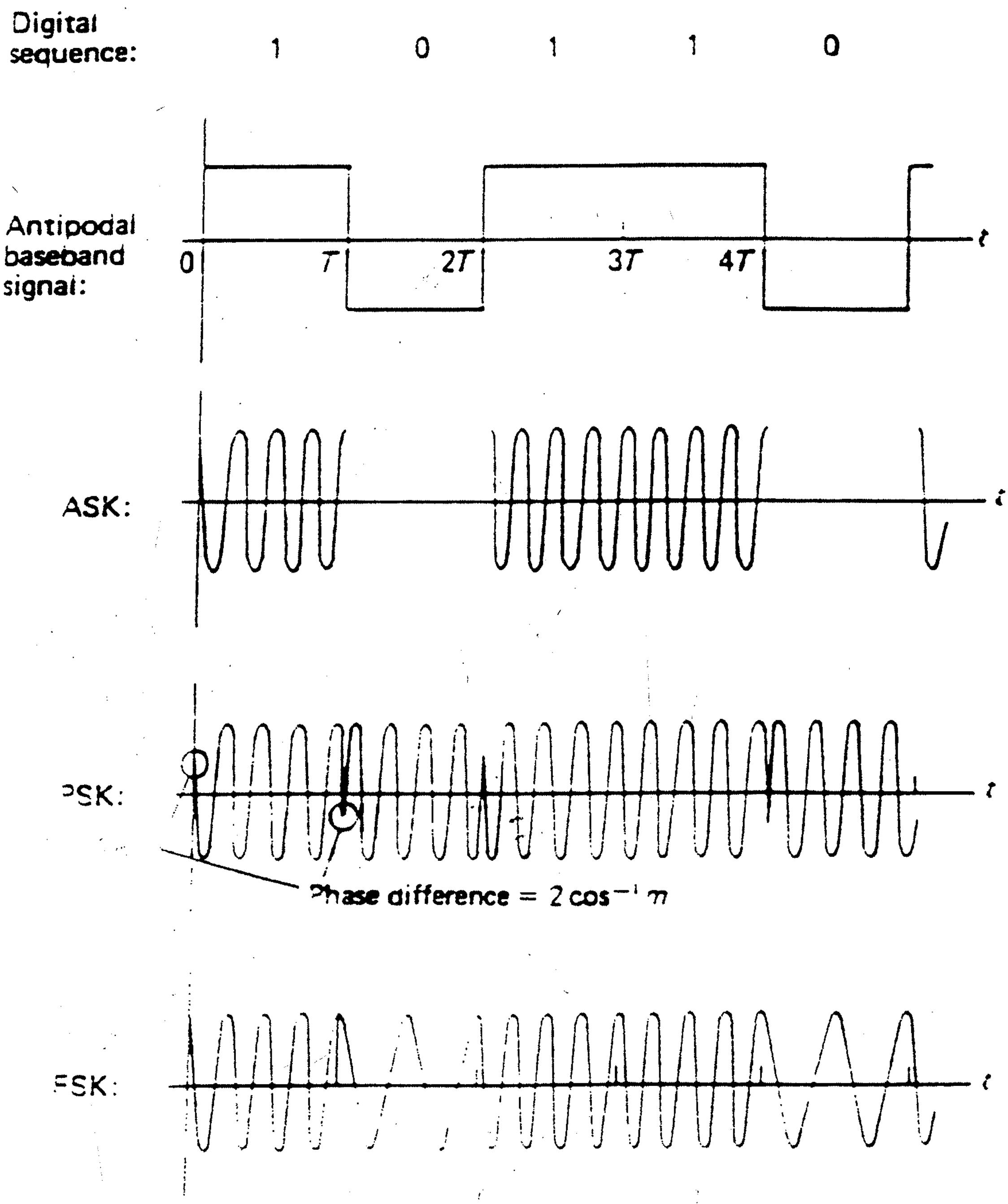


FIGURE 7.14 Waveforms for ASK, PSK, and FSK modulation

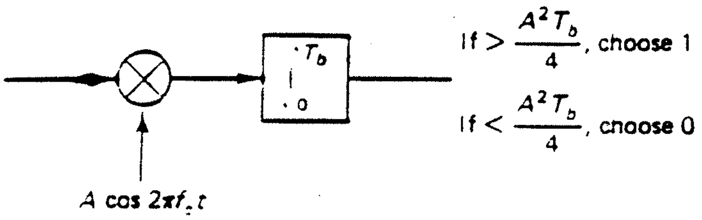


Figure 8.43 Matched filter detector for OOK.

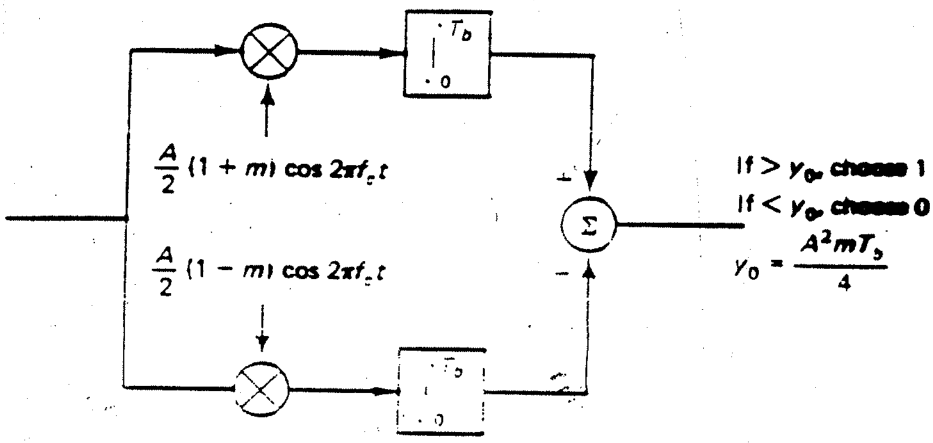


Figure 8.42 Matched filter detector for BASK.

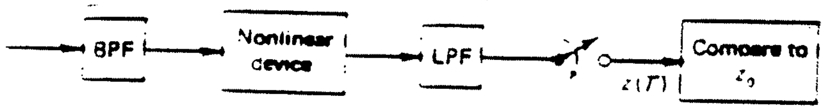
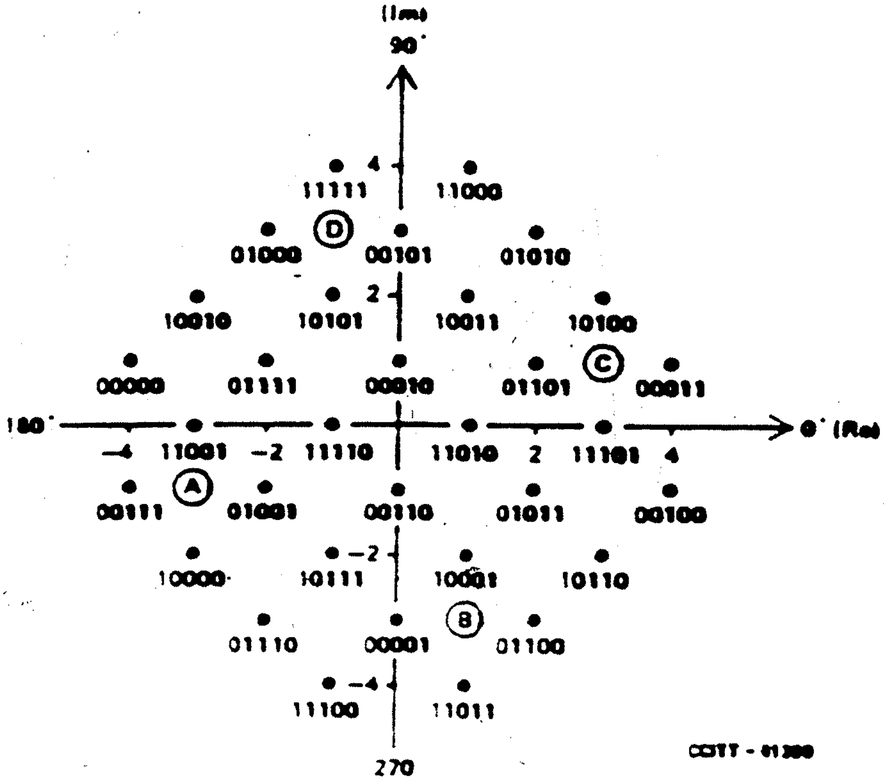


Figure 8.44 Envelope detector for OOK BASK.



Los numeros binarios representan $Y0_n\ Y1_n\ Y2_n\ Q3_n\ Q4_n$

FIGURA 3/V.32

Constelación de señal de 32 puntos con codificación en rejilla para 9600 bit/s y subconjunto de estados A, B, C y D utilizados a 4800 bit/s y para el acondicionamiento

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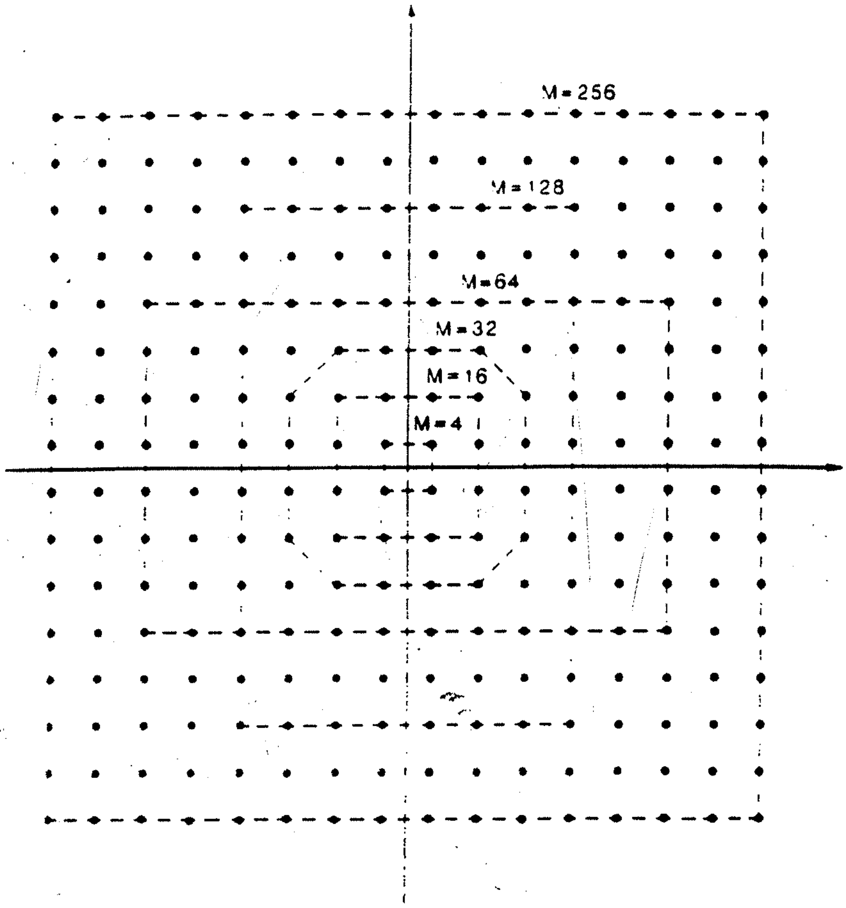


Figure 5.22 Rectangular AM–PM signal constellations. The dotted lines delimit the region of the signal points pertaining to that particular M. Notice the cross constellations for $M = 32 = 2^5$ and $M = 128 = 2^7$.

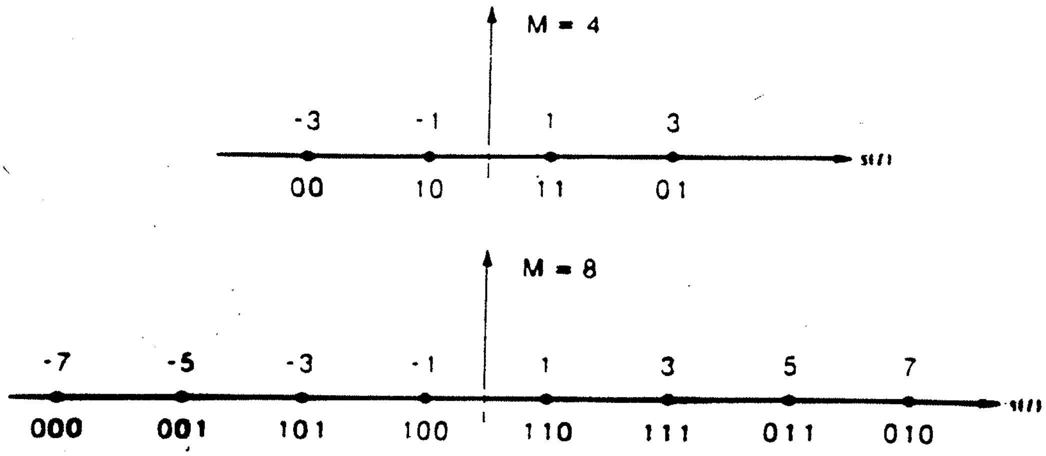
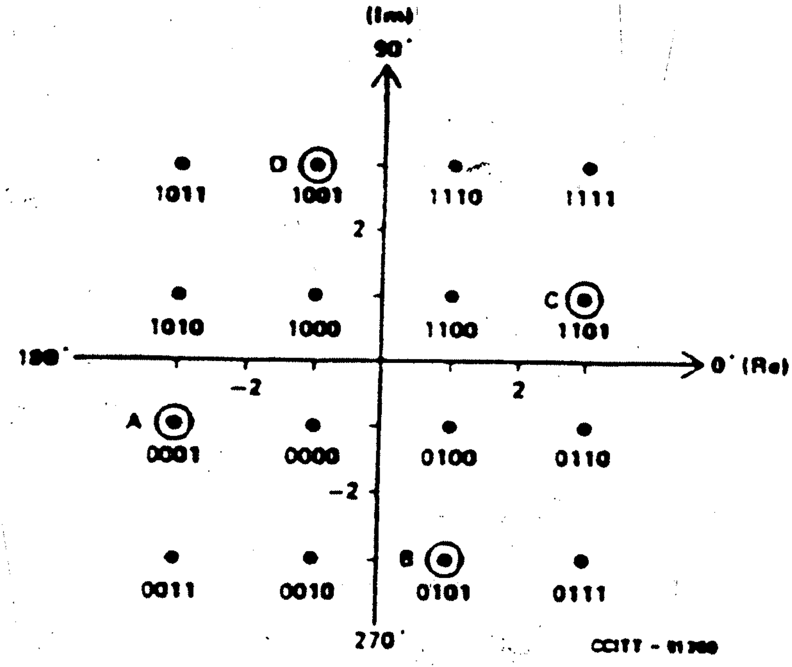


Figure 5.2 Geometrical representation of PAM signal sets.



Los numeros binarios representan $Y1_n\ Y2_n\ Q3_n\ Q4_n$

FIGURA 1/V.32

Constelación de señal de 16 puntos con codificación no redundante para 9600 bit/s y subconjunto A, B, C y D de estados utilizados a 4800 bit/s y para el acondicionamiento

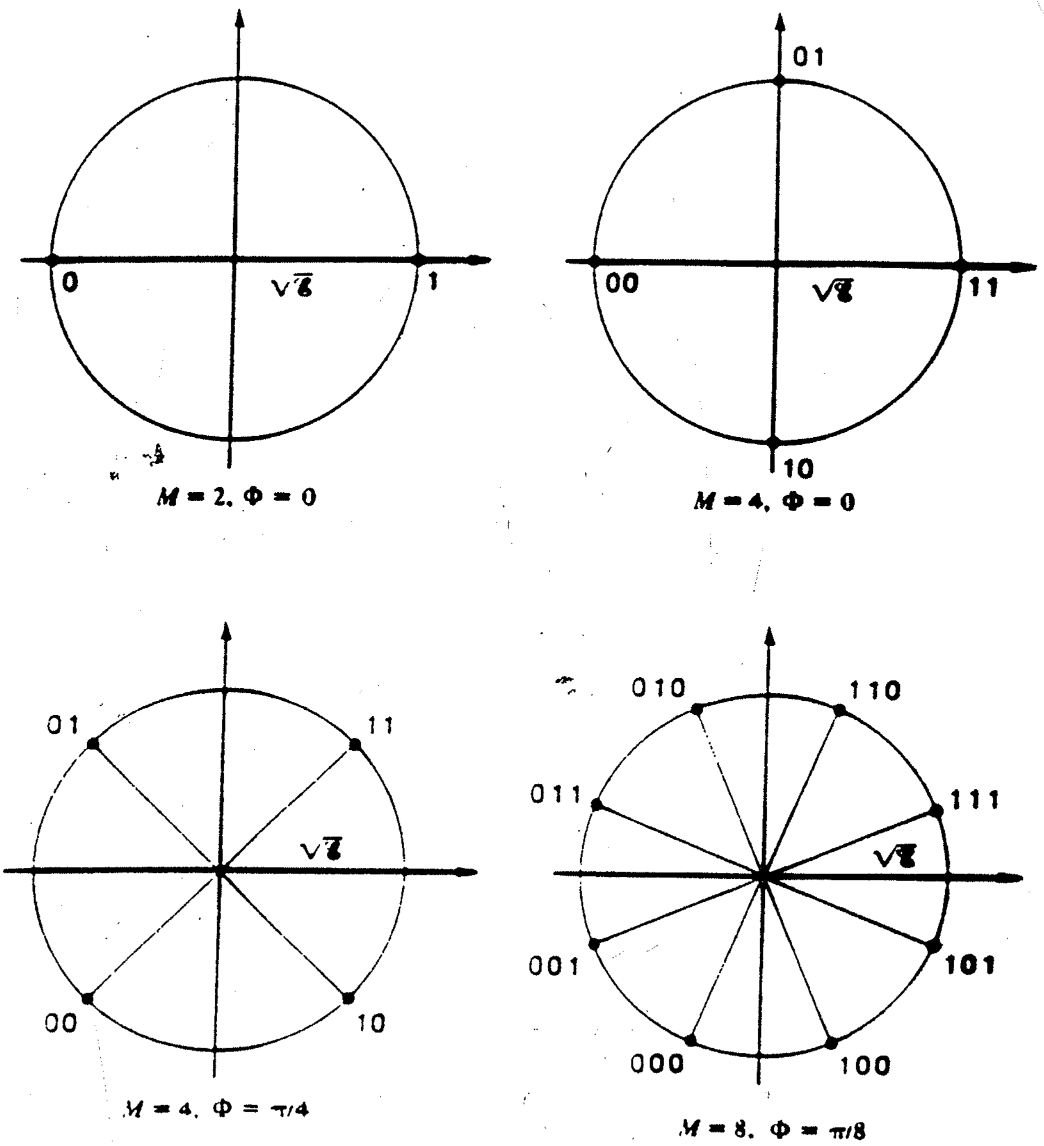


Figure 5.5 Geometrical representation of PSK signal sets.

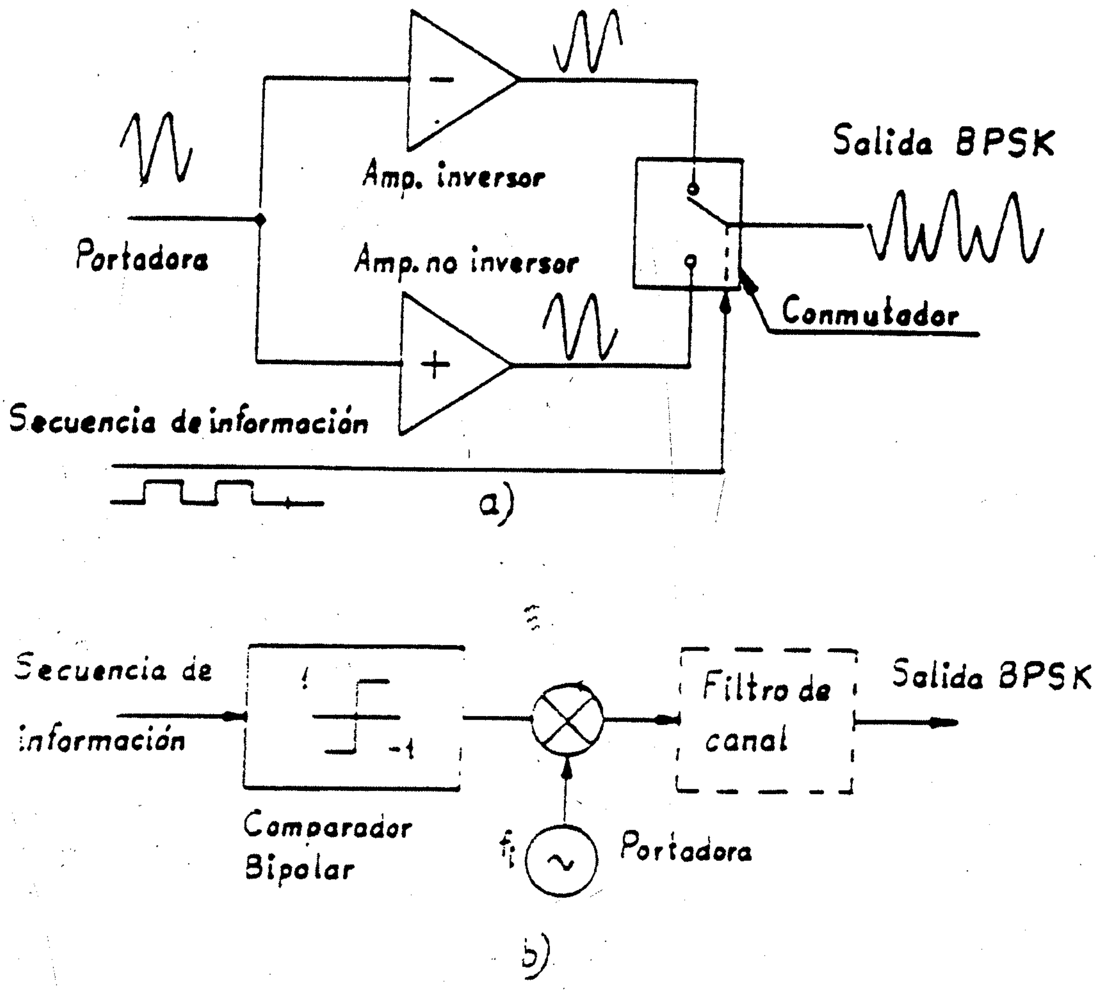


Figura 8.12. Moduladores BPSK.

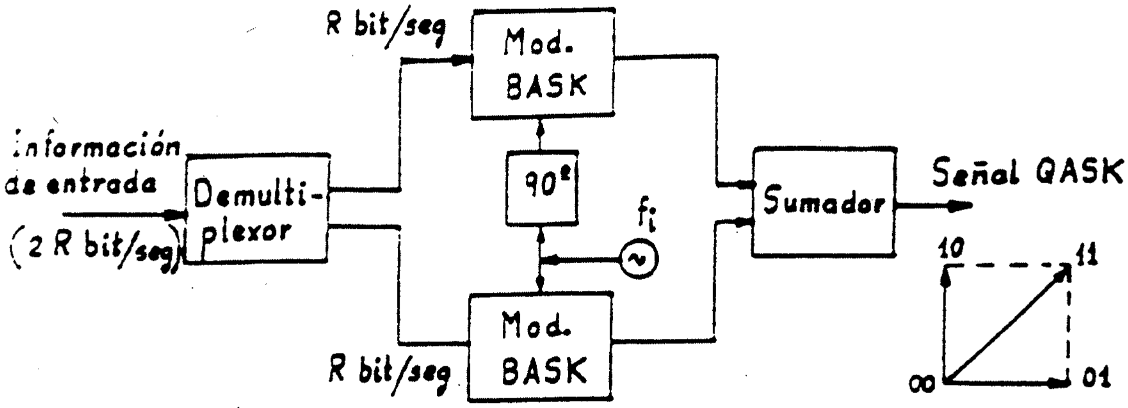


Figura 8.6. Modulador de QASK.

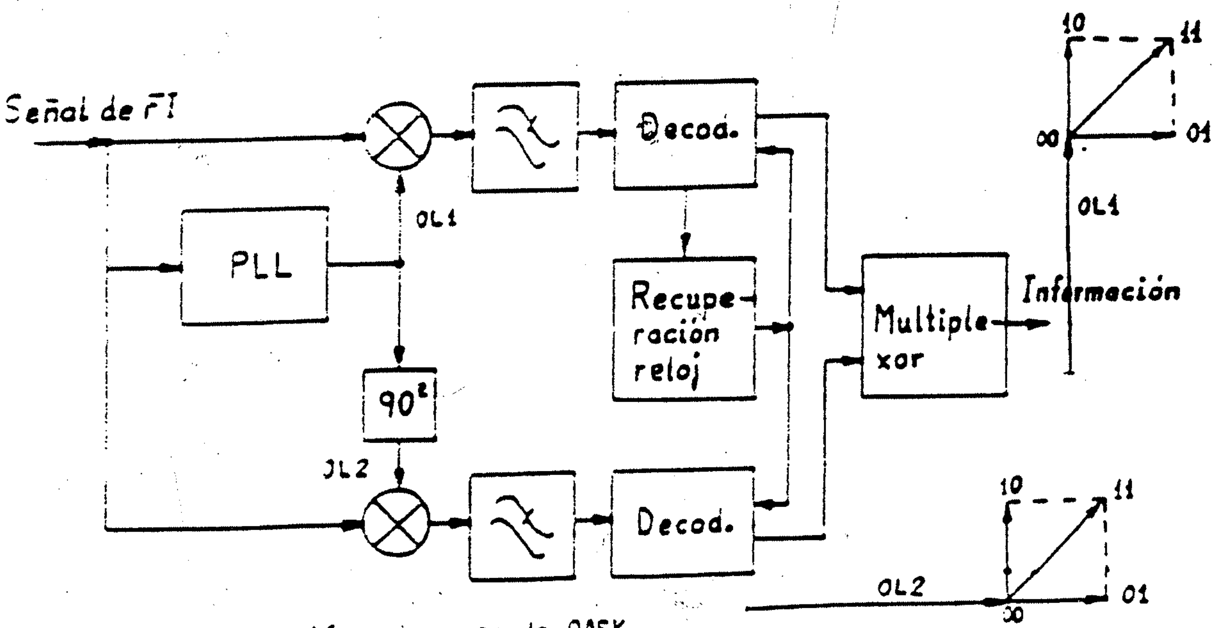


Figura 8.10. Detección coherente de QASK.

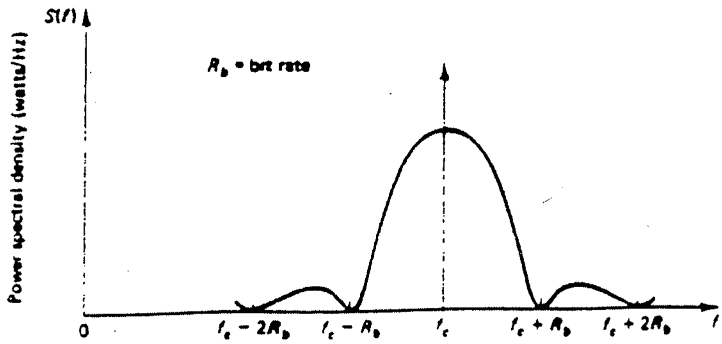


Figure 8.4 Power spectral density of random BASK signal

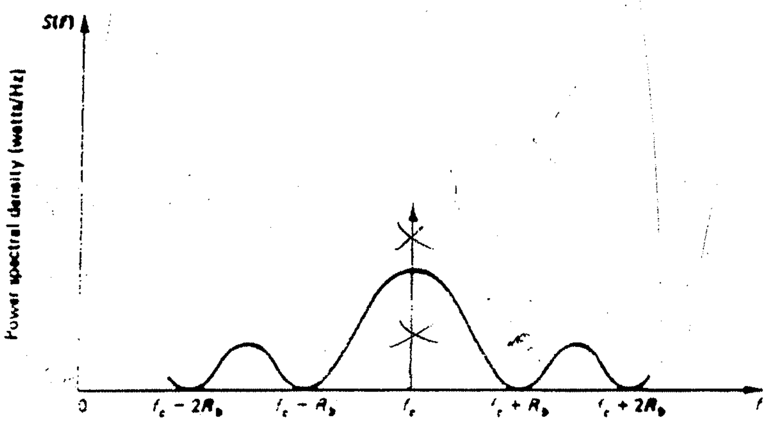


Figure 10.3 Power spectral density of random BPSK waveform

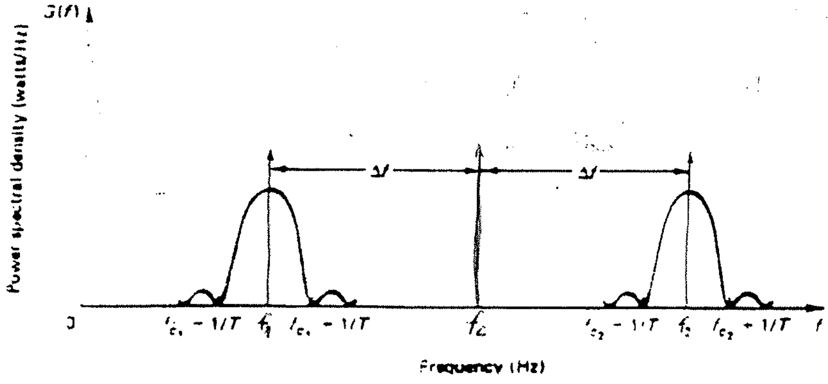


Figure 9.5 Power spectral density of random FSK waveform

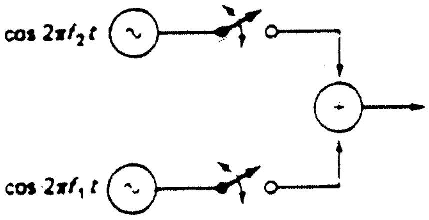


Figure 9.4 FSK as the superposition of two ASK waveforms

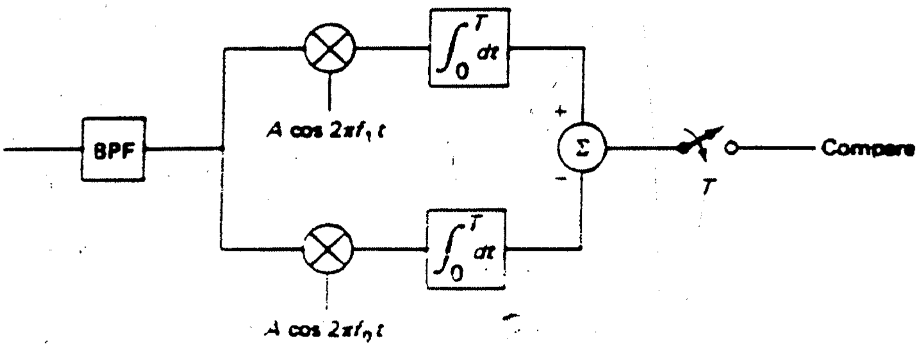


Figure 9.11 Coherent FSK detector

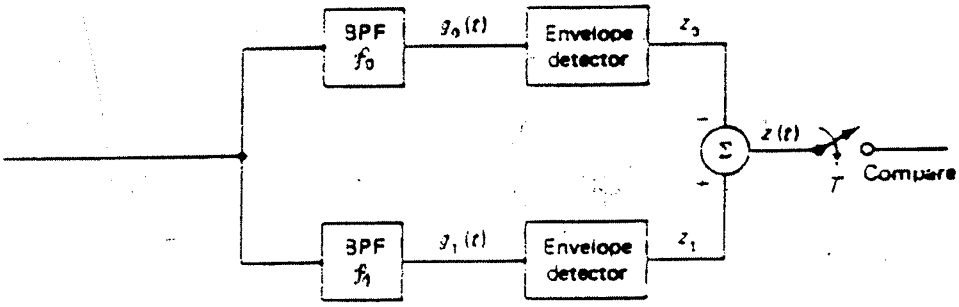


Figure 9.12 Incoherent FSK detector

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